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Metastability range of the Bose condensate in ^{7}Li-fermion mixtures at T = 0

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Abstract. We evaluate the ground state of a mixture of bosons and spin-polarized fermions in the case of attractive boson-boson interactions, using a variational Ansatz for the Bose condensate wave function and the Thomas-Fermi approximation for the fermions in the mean field of the condensate. Within this approximation we show that the presence of the fermions tends to restrict the metastability range of the condensate, irrespectively of the sign of the boson-fermion interactions. Numerical illustrations are reported for mixtures of ^{7}Li atoms with fermions having the ^{6}Li mass.

PACS. 03.75.Fi Phase coherent atomic ensembles; quantum condensation phenomena – 67.40.Db Quantum statistical theory; ground state, elementary excitations

The report of Bose-Einstein condensation in a vapour of spin-polarized ^{7}Li atoms which are confined and cooled inside a permanent-magnet trap [1,2] has drawn special interest from the fact that the s-wave scattering length of these atoms is negative [3]. In these conditions a macroscopic Bose condensate is predicted to be mechanically unstable against collapse [4]. However, in the inhomogeneous state which is realized inside the trap the zero-point energy can exceed the attractive interactions and prevent collapse of the condensate when it contains a finite number N of atoms up to a critical number N_c [5]. For $N < N_c$ the energy of the condensate as a function of its central density shows a metastable minimum separated from collapse by a barrier, suggesting the possibility of macroscopic quantum tunneling [6]. Thermal fluctuations decrease the number of ^{7}Li condensate particles at the point of instability [7].

Trapping of fermionic species has also been achieved for ^{6}Li [8] and ^{40}K [9], and trapped mixtures of bosonic and fermionic species are expected to become accessible to experiment in the near future. Illustrative calculations for the case of repulsive boson-boson and boson-fermion interactions have been reported by Mølmer [10] in the Thomas-Fermi approximation at zero temperature and by Amoruso *et al.* [11] in a semiclassical three-fluid model at finite temperature. In the present work we evaluate the ground state of boson-fermion mixtures in the case of attractive boson-boson interactions, with specific attention to the role of the fermionic component in modifying the range of metastability of the Bose condensate.

We assume that the vapour has reached equilibrium inside the trap and take the fermionic component as a dilute, spin-polarized Fermi gas. The s-wave collisions between pairs of fermions are inhibited by the Pauli principle, so that to leading order only p-wave scattering and dipole-dipole magnetic interactions play a role. However, these interactions have very small effects at very low temperature [11,12]. We neglect them and introduce the boson-boson and boson-fermion coupling parameters $g = 4\pi\hbar^2 a_b/m_b$ and $f = 2\pi\hbar^2 a_f/m_r$, with a_b and a_f the corresponding s-wave scattering lengths, m_b and m_f the atomic masses and $m_r = m_b m_f/(m_b + m_f)$ the reduced mass of a boson-fermion pair. We adopt the Thomas-Fermi approximation for the fermion density profile $n_f(\mathbf{r})$ in the mean field of the condensate, that is

$$n_f(\mathbf{r}) = \frac{1}{6\pi^2}(2m_f/\hbar^2)^{3/2}[\varepsilon_f - V_{\rm ext}^{(f)}(\mathbf{r}) - f|\Phi(\mathbf{r})|^2]^{3/2} \quad (1)$$

where ε_f is the Fermi energy, $V_{\rm ext}^{(f)}(\mathbf{r})$ is the confining potential acting on the fermions and $\Phi(\mathbf{r})$ is the wave function of the Bose condensate. The Fermi energy is to be determined from the number N_f of fermions in the trap according to $N_f = \int d\mathbf{r} n_f(\mathbf{r})$. From previous analyses of the semiclassical regime and finite-size effects in Fermi vapours [13], equation (1) should be a good approximation for the present purposes.

The Gross-Pitaevskii equation for the condensate wave function $\Phi(\mathbf{r})$ is

$$\Big[-\frac{\hbar^2}{2m_b}\nabla^2 + V_{\rm ext}^{(b)}(\mathbf{r}) + g|\Phi(\mathbf{r})|^2 + f n_f(\mathbf{r})\Big]\Phi(\mathbf{r}) = \mu_b\Phi(\mathbf{r}) \quad (2)$$

where $V_{\rm ext}^{(b)}(\mathbf{r})$ is the confining potential acting on the bosons and the chemical potential μ_b is to be determined from the number N_b of bosons in the trap according

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to $N_\mathrm{b} = \int \mathrm{d}\mathbf{r}|\Phi(\mathbf{r})|^2$. Since $n_\mathrm{f}(\mathbf{r})$ is a function of $\Phi(\mathbf{r})$ (see (1)), the fermion-boson interaction term in (2) is a non-linear one and reflects, of course, the sign of the coupling parameter f. Bearing this in mind, we construct the energy functional $E[\Phi]$ from which (2) can be obtained by minimization. This is

$$E[\Phi] = \int \mathrm{d}\mathbf{r}\Big[\frac{\hbar^2}{2m_\mathrm{b}}|\nabla\Phi|^2 + V_\mathrm{ext}^{(\mathrm{b})}|\Phi|^2 + \frac{1}{2}g|\Phi|^4 - \frac{1}{15\pi^2}\Big(\frac{2m_\mathrm{f}}{\hbar^2}\Big)^{3/2}(\varepsilon_\mathrm{f} - V_\mathrm{ext}^{(\mathrm{f})} - f|\Phi|^2)^{5/2}\Big]. \quad (3)$$

In the following we take isotropic confinements for both components, *i.e.* $V_\mathrm{ext}^{(\mathrm{b})}(\mathbf{r}) = (1/2)m_\mathrm{b}\omega_\mathrm{b}^2 r^2$ and $V_\mathrm{ext}^\mathrm{f}(\mathbf{r}) = (1/2)m_\mathrm{f}\omega_\mathrm{f}^2 r^2$. We then assume a Gaussian Ansatz for the condensate density,

$$|\Phi(y)|^2 = N_\mathrm{b}\left(\frac{2\alpha}{\pi}\right)^{3/2}\exp(-2\alpha y^2) \quad (4)$$

and determine the parameter α variationally from the energy functional in (3). In (4) we have set $r = y a_\mathrm{ho}$ with $a_\mathrm{ho} = (\hbar/m_\mathrm{b}\omega_\mathrm{b})^{1/2}$.

Upon inserting (4) into (3) we find that the energy becomes a function $E(\alpha)$ of the variational parameter α, given by

$$E(\alpha) = \frac{3}{2}N_\mathrm{b}\left(\alpha + \frac{1}{4\alpha}\right) + \frac{1}{2}gN_\mathrm{b}^2\left(\frac{\alpha}{\pi}\right)^{3/2} - \frac{4}{15\pi}\left(2\frac{m_\mathrm{f}}{m_\mathrm{b}}\right)^{3/2}\int_0^\infty y^2 F(y;\alpha)\mathrm{d}y \quad (5)$$

where

$$F(y;\alpha) = \left[\varepsilon_\mathrm{f} - N_\mathrm{b} f\left(\frac{2\alpha}{\pi}\right)^{3/2}\exp(-2\alpha y^2) - \frac{m_\mathrm{f}\omega_\mathrm{f}^2}{2m_\mathrm{b}\omega_\mathrm{b}^2}y^2\right]^{5/2}. \quad (6)$$

In these equations energies are in units of $\hbar\omega_\mathrm{b}$ and the coupling constant f is in units of $\hbar\omega_\mathrm{b}a_\mathrm{ho}^3$.

Given the values of the system parameters (atomic masses, scattering lengths and trap frequencies) and the number N_f of fermions, the function $E(\alpha)$ in (5) has a minimum at $\alpha = \alpha_\mathrm{eq}$ for all values of the number N_b of trapped bosons which are below a critical value N_c. The value of the chemical potential μ_b is obtained for $N_\mathrm{b} < N_\mathrm{c}$ from $\mu_\mathrm{b} = \partial E(\alpha)/\partial N_\mathrm{b}|_{\alpha=\alpha_\mathrm{eq}}$. Clearly, the minimization of $E(\alpha)$ must be carried out in parallel with the evaluation of the fermion density profile from (1) and with the determination of the Fermi energy from the normalization of the fermion density profile to N_f. As the final result one obtains the density profiles of both fermions and bosons at equilibrium, the bosonic one being given by (4) with $\alpha = \alpha_\mathrm{eq}$.

At $N_\mathrm{b} = N_\mathrm{c}$ the minimum in $E(\alpha)$ becomes a flex point and the vapour becomes unstable against collapse. The critical number N_c of bosons and the corresponding value α_c of the condensate width parameter are therefore determined by asking that the first and the second derivative of $E(\alpha)$ with respect to α should vanish at $\alpha = \alpha_\mathrm{c}$ and $N_\mathrm{b} = N_\mathrm{c}$. These two conditions are easily derived in explicit form from equations (5, 6) and are solved numerically for α_c and N_c in parallel with the evaluation of the fermion density profile and of the Fermi energy ε_f .

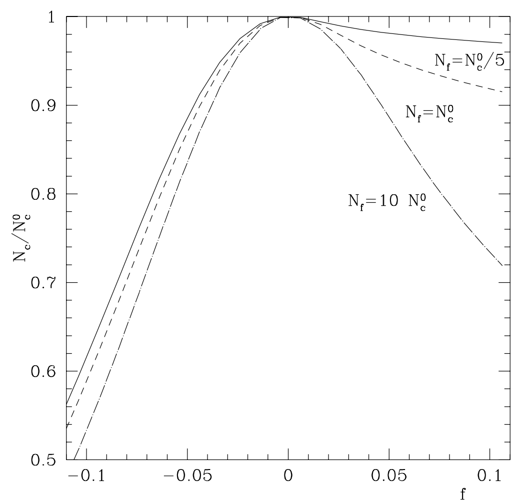

Fig. 1. Change in the reduced critical number $N_\mathrm{c}/N_\mathrm{c}^0$ of bosons in the condensate *versus* the reduced fermion-boson coupling strength f, for three values of the reduced number $N_\mathrm{f}/N_\mathrm{c}^0$ of fermions. The system parameters refer to ^{7}Li atoms in a mixture with fermions having the mass of ^{6}Li atoms, the two vapours being confined in spherically symmetric harmonic traps with $\omega_\mathrm{b} = \omega_\mathrm{f} = 908\ \mathrm{s}^{-1}$.

Figure 1 reports our result for the ratio $N_\mathrm{c}/N_\mathrm{c}^0$ over a very wide range of values for the boson-fermion coupling parameter f (in units of $\hbar\omega_\mathrm{b}a_\mathrm{ho}^3$) at three values of the ratio $N_\mathrm{f}/N_\mathrm{c}^0$, N_c^0 being the critical number of bosons at $N_\mathrm{f} = 0$. The parameters describing the bosonic component are those appropriate to ^{7}Li atoms in the trap realized by Bradley *et al.* [1] ($a_\mathrm{b} = -27.6$ Bohr radii, $\omega_\mathrm{b} = 908\ \mathrm{s}^{-1}$ and m_b = mass of ^{7}Li atom), while for the fermions we have chosen $\omega_\mathrm{f} = \omega_\mathrm{b}$ and m_f = mass of ^{6}Li atom. The ^{6}Li–^{7}Li interaction has been evaluated by Abraham *et al.* [3], with the result $a_\mathrm{f} = 40.9$ Bohr radii or $f = 0.0093$ in the reduced units used in Figure 1.

It is clear from Figure 1 that the addition of fermions restricts the range of metastability of the ^{7}Li condensate, irrespectively of the sign of the fermion-boson coupling and indeed most drastically in the case of a negative fermion-boson scattering length. However, only the central portion of the curves in Figure 1 is of interest when the fermion-boson scattering length and the fermion mass are comparable in magnitude with the bosonic ones. In these conditions, which apply to the ^{6}Li–^{7}Li mixture, the effect of the fermions on the metastability of the condensate is small.

Although the results shown in Figure 1 refer to a specific choice of system parameters, their general aspects can be understood from the density profiles that we report at $N_\mathrm{f}/N_\mathrm{c}^0 = 1$ in Figures 2 and 3 for $f > 0$ and $f < 0$, respectively. The case of fermion-boson repulsion (with the choice $a_\mathrm{f} = 40.9$ Bohr radii or $f = 0.0093$) is illustrated in Figure 2. The Fermi vapour tends to spread

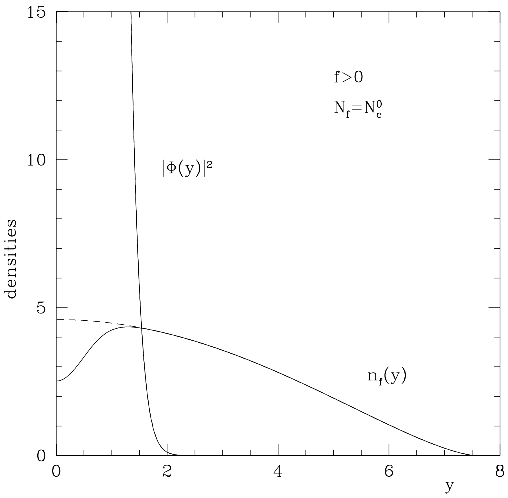

Fig. 2. Density profiles of bosons and fermions (full curves) at $N_\mathrm{b} = N_\mathrm{c}$, for the case $N_\mathrm{f}/N_\mathrm{c}^0 = 1$ and $f = 0.0093$ (corresponding to the ^{6}Li–^{7}Li mixture). The dashed curve shows the profile for an independent Fermi gas ($f = 0$).

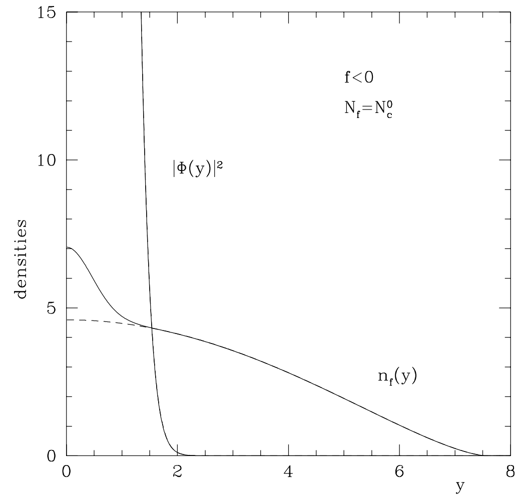

Fig. 3. The same as in Figure 2, but for $f = -0.0093$.

out under its kinetic pressure and this process is enhanced by the fermion-boson repulsion, as is also the case for boson-fermion mixtures with repulsive boson-boson interactions [10,11]. In the present case of boson-boson attractions, however, the Bose condensate is very compact and is squeezed inward by the repulsions from the surrounding Fermi vapour. This squeezing of the condensate is the cause for the decrease in its range of metastability which is seen in the RHS part of Figure 1.

Figure 3 illustrates the density profiles in the case of attractive boson-fermion interactions with the choice $a_\mathrm{f} = -40.9$ Bohr radii or $f = -0.0093$. The dip shown in Figure 2 in the fermion density near the centre of the condensate is replaced in Figure 3 by a fermion density pile-up induced by the fermion-boson attractions. This feature of the fermion density profile is the cause for the rapid decrease of the metastability range of the condensate which is seen in the left side of Figure 1.

In summary, we have studied the metastability of a Bose condensate with attractive effective interactions in the presence of a degenerate Fermi gas at zero temperature. The main result is that the metastability range of the condensate in a boson-fermion mixture with attractive boson-boson interactions is reduced for both repulsive and attractive fermion-boson interactions, this being related to the features of the density profiles of the two components. The effect is only minor when the two components have comparable weights in terms of atom numbers, atomic masses and scattering lengths, these conditions being apparently satisfied in the ^{6}Li–^{7}Li mixture for $N_\mathrm{f} \approx N_\mathrm{b}$. However, the addition of even a relatively small number of fermions may suffice to destroy a Bose condensate with attractive interactions if $|f| \gg |g|$. In such critical situations our study should be supplemented by a fully quantitative evaluation from the coupled non-linear Schrödinger equations for boson and fermion vapours, after a specific quantitative assessment of the system parameters.

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